

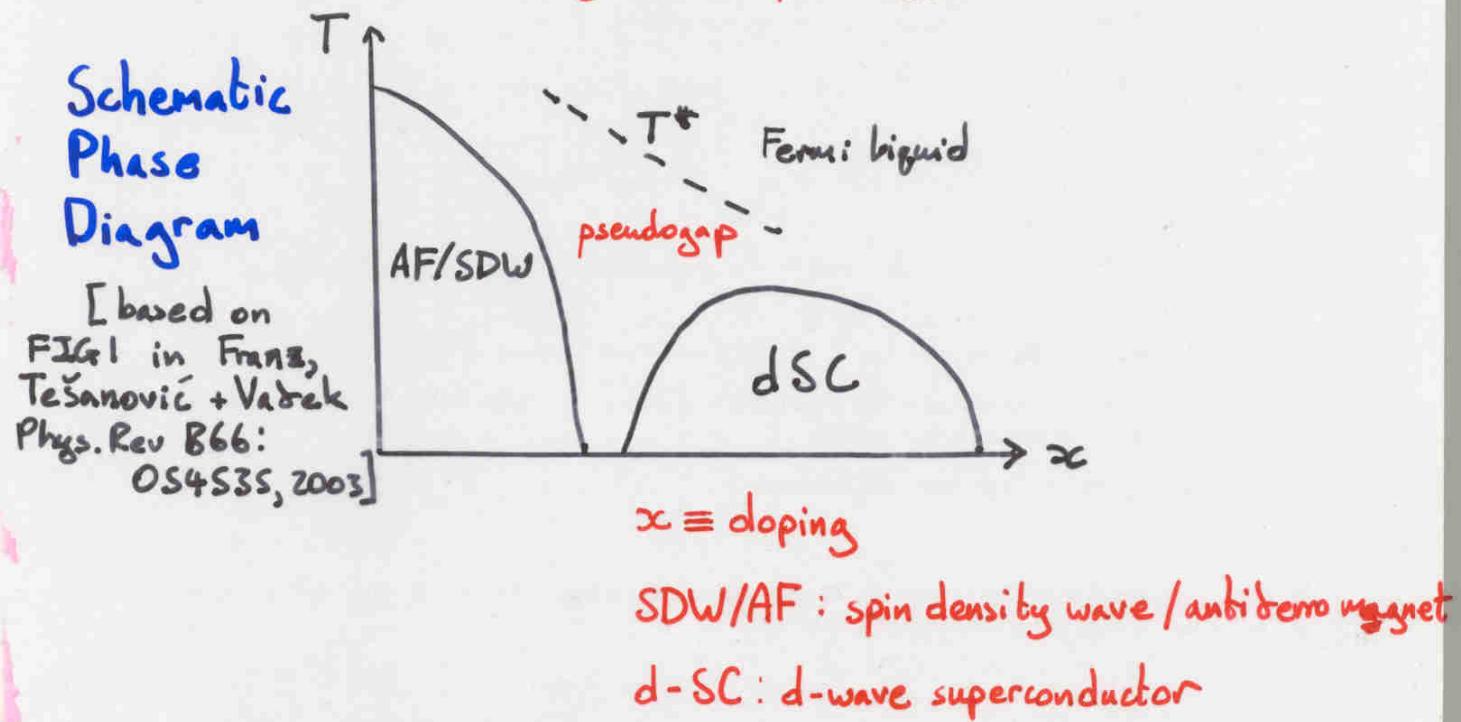
QED IN 2+1 DIMENSIONS WITH FERMI AND GAP ANISOTROPIES

Jorwerth Thomas
Simon Hands

University of Wales Swansea

Background: The phase diagram of cuprate superconductors

- The cuprates are a family of superconductors with a number of interesting properties.
- They exhibit superconducting behaviour at high temperatures
- They have a pseudogap phase



pseudogap: phase where an energy gap exists without the corresponding quasiparticle excitation.

- Study of this phase may give us insight into the structure of the entire phase diagram
- Recent approaches (Tešanović et al.; Balents, Fisher and Nayak; Herbut) emphasize the study of the transition from d-SC to pseudogap
- As we shall see, the question of chiral symmetry breaking in QED-3 is of critical importance to the approach we are taking

[Refs: Balents, Fisher, Nayak Phys. Rev. B 60, 1654 (1999)
Herbut Phys. Rev. B 66, 094504 (2002)]

Follows
Herbut
2002

d-wave S-C \longrightarrow QED₃ effective theory

d-wave quasiparticle action at T=0:

$$S = T \sum_{\underline{k}, \sigma, \omega_n} \left[(i\omega_n - \epsilon_{\underline{k}}) c_{\sigma}^{\dagger}(\underline{k}, \omega_n) c_{\sigma}(\underline{k}, \omega_n) - \frac{\sigma}{2} \Delta(\underline{k}) c_{\sigma}^{\dagger}(\underline{k}, \omega_n) c_{-\sigma}^{\dagger}(-\underline{k}, -\omega_n) + h.c. + O(c^4) \right]$$

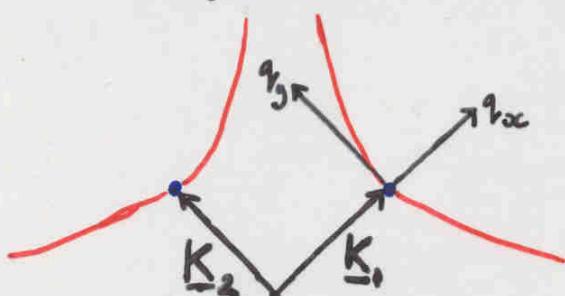


diagram of
Fermi Surface
(redline) +
nodes (blue dots)

Assume 2 spatial dimensions
(cuprate SC is layered
thinly)

$c_{\sigma}^{\dagger}, c_{\sigma}$: e^- -operators

$\sigma = +, -$: spin

ω_n : Fermionic Matsubara frequency

$O(c^4)$: short-range quasiparticle interactions

Rewrite S in terms of the two fields:

$$\Psi_i^{'+}(\underline{q}, \omega_n) = (c_+(\underline{k}, \omega_n), c_-(-\underline{k}, -\omega_n), c_+^{\dagger}(\underline{k} - \underline{Q}_i, \omega_n), c_-^{\dagger}(-\underline{k} + \underline{Q}_i, \omega_n))$$

$i = 1, 2$; $\underline{Q}_i = 2\underline{K}_i$ (wave vector connecting diagonal nodes)

where for a small momentum $\underline{q} \Rightarrow \underline{k} = \underline{K}_i + \underline{q}$, $|\underline{q}| \leq |\underline{K}_i|$
we can linearise:

$$\gamma_{\underline{k}} = v_F q_{\perp x} + O(q^2) \quad \Delta_{\underline{k}} = v_{\Delta} q_{\perp y} + O(q^2)$$

$\underbrace{\qquad}_{\text{Fermi velocity}}$ $\underbrace{\qquad}_{\text{gap velocity}}$

And so derive: $\Rightarrow \Rightarrow$

$$S[\Psi_i'] = \int d^2 \underline{r} \int_0^B d\varepsilon \bar{\Psi}_i' [\gamma_0 \partial_{\varepsilon} + \gamma_1 v_F \partial_{\perp x} + \gamma_2 v_{\Delta} \partial_{\perp y}] \Psi_i' + (1 \rightarrow 2, x \leftrightarrow y) + O(\partial \bar{\Psi} \partial \Psi')$$

$$\gamma_0 = \sigma_1 \otimes I \quad \gamma_1 = \sigma_2 \otimes \sigma_3 \quad \gamma_2 = \sigma_2 \otimes \sigma_1$$

If we write: $\delta = \sqrt{v_F v_\Delta}$, $B = \frac{v_F}{\sqrt{\Delta}}$

(Lee and Hwang, Phys. Rev. B66: 094512 (2002))

$$v_F \partial_x \rightarrow \frac{\delta}{\sqrt{B}} \partial_x \quad v_\Delta \partial_y \rightarrow \delta \sqrt{B} \partial_y$$

Gauge field: dSC order parameter has phase degree of freedom

→ formation of vortices

→ condensation of these vortices
disorders dSC phase → pseudogap phase

→ Goldstone boson, modelled as a photon field minimally coupled to the photon fermion

[Motrunich, Vishwanath arXiv: cond-mat/0311222

Kouwen, Rosenstein, Nuc Phys B350, 325
Elizer (1991)

Mod Phys Lett A 5,
2733 (1990)]

$$\rightarrow S = \int d^2 r \, d\tau \left(\bar{\Psi}_1 \left[\delta_0 (\partial_\tau + i a_0) + \frac{\delta}{\sqrt{B}} \delta_1 (\partial_x + i a_1) + \delta \sqrt{B} \delta_1 (\partial_y + i a_2) \right] \Psi_1 + (1 \rightarrow 2, x \leftrightarrow y) + \frac{1}{2e^2} F_{\mu\nu} F_{\mu\nu} \right)$$

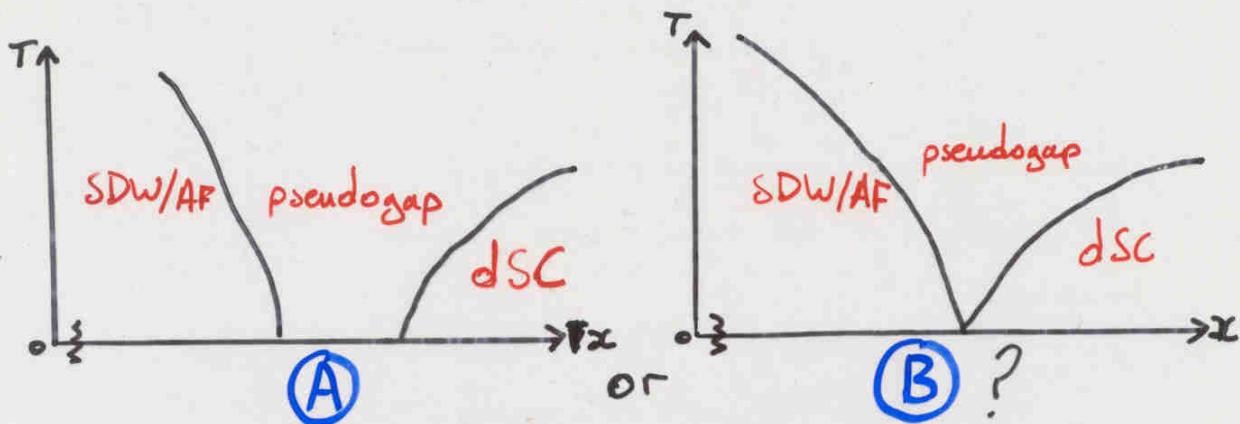
- Low energy effective theory for pseudogap phase
→ QED₃ with anisotropies

- Doping ω is proportional to κ

@ optimum doping, $\kappa \sim 10$

Chiral Symmetry Breaking

What does the phase diagram look like at $T=0$?



- Formation of chiral condensate → signals onset of SDW
[E Herbut, Tešanović et al.]
becomes small AF phase as x decreases
- Chirally symmetric phase can be identified with the presence of a pseudogap

→ What is the critical # of flavours (N_c) for chiral symmetry breaking in QED_3 ?

Schwinger-Dyson methods: ambiguous ($N_c = \infty, \frac{32}{\pi^2}, \approx 4$) [†]

Appelquist's thermodynamic argument: $N_c < \frac{3}{2}^*$

Correspondence with Thirring model: $N_c > 4$ [‡]

Lattice calculations: inconclusive [†]

Is this also true for anisotropic simulations?

* Appelquist, Cohen, Schmitz Phys Rev D60 045003 (1999)

† Various References given in [†]

‡ Hands, Kogut, Scorzato, Strouthos arXiv:hep-lat/0404013

The Model

$$S = \frac{\beta}{2} \sum_{x, \mu \in \nu} \Theta_{\mu\nu}(x) \Theta_{\mu\nu}(x) + \sum_{i=1}^N \sum_{x, x'} \bar{\chi}_i(x) M(x, x') \chi_i(x')$$

Gauge field: $\Theta_{\mu\nu}(x) = \Theta_{x\mu} + \Theta_{x+\hat{\mu}, \nu} - \Theta_{x+\hat{\nu}, \mu} - \Theta_{x\nu}$

$\Theta_{x\mu}$ is a photon field defined on a link

$$M(x, x') = m_0 \delta_{x, x'} + \frac{1}{2} \sum_{\mu=1}^3 \xi_\mu(x) \left[\delta_{x', x+\hat{\mu}} U_{x\mu} - \delta_{x', x-\hat{\mu}} U_{x\mu}^* \right]$$

$$U_{x\mu} \equiv \exp(i\Theta_{x\mu}) \quad U_{x+\hat{\mu}, -\mu} = U_{x\mu}^*$$

$$\xi_\mu(x) = \lambda_\mu \eta_{\mu\mu}(x)$$

$$\eta_\mu(x) = (-1)^{x_1 + \dots + x_{\mu-1}}$$

$$\lambda_1 = K^{-1/2}$$

$$\lambda_2 = K^{+1/2}$$

$$\lambda_3 = 1$$

These describe the anisotropy with $\delta=1$

- Note: Θ^2 is unbounded from above \rightarrow noncompact formulation

Simulation Notes

- Hybrid Monte-Carlo simulation of Grassmann staggered fermion fields on 16^3 lattice

- Even-odd partitioning implemented

$$\rightarrow N=1 \Rightarrow N_f=2 \text{ in continuum limit}$$

Measurements

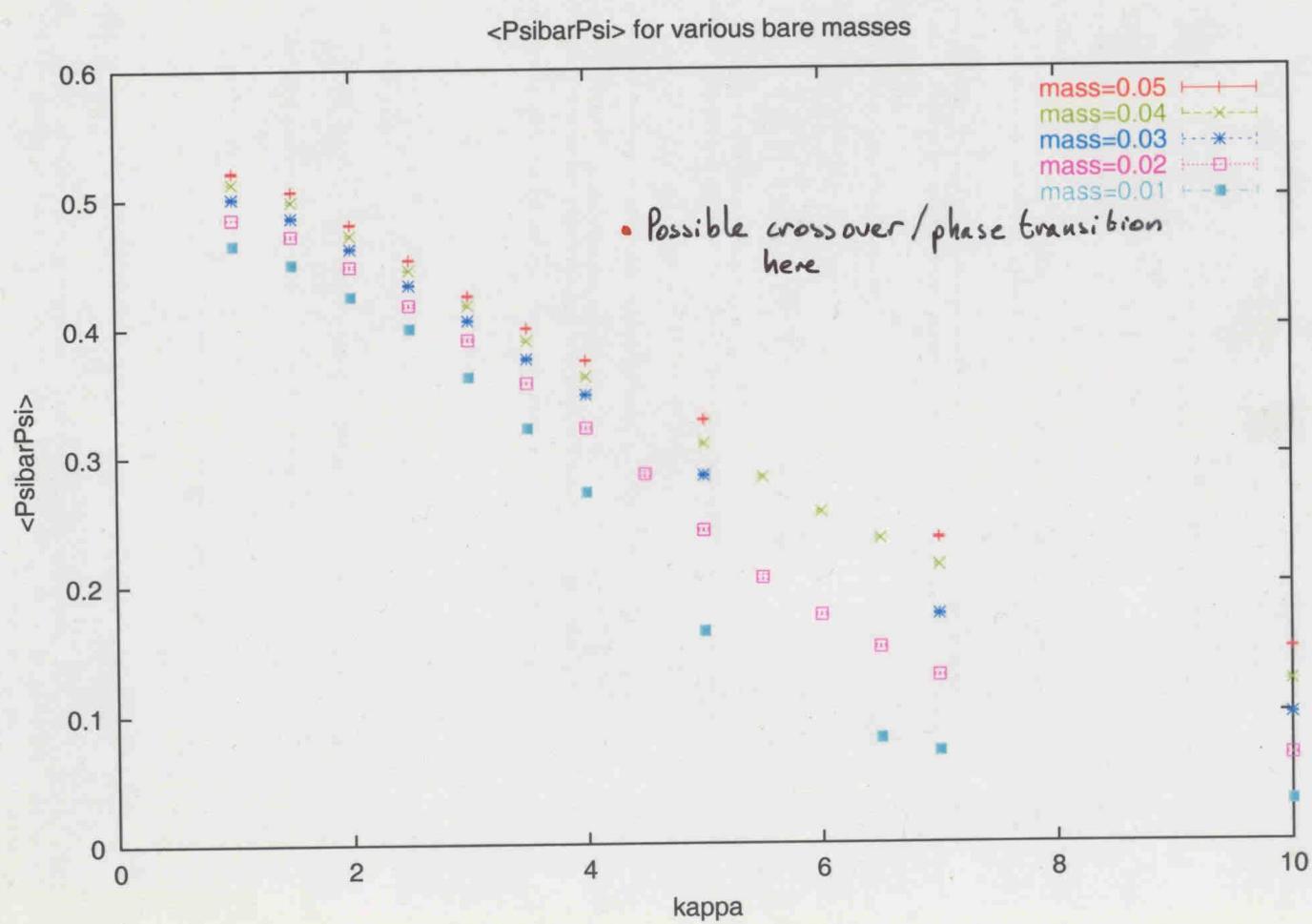
- Chiral Condensates

- Pion mass

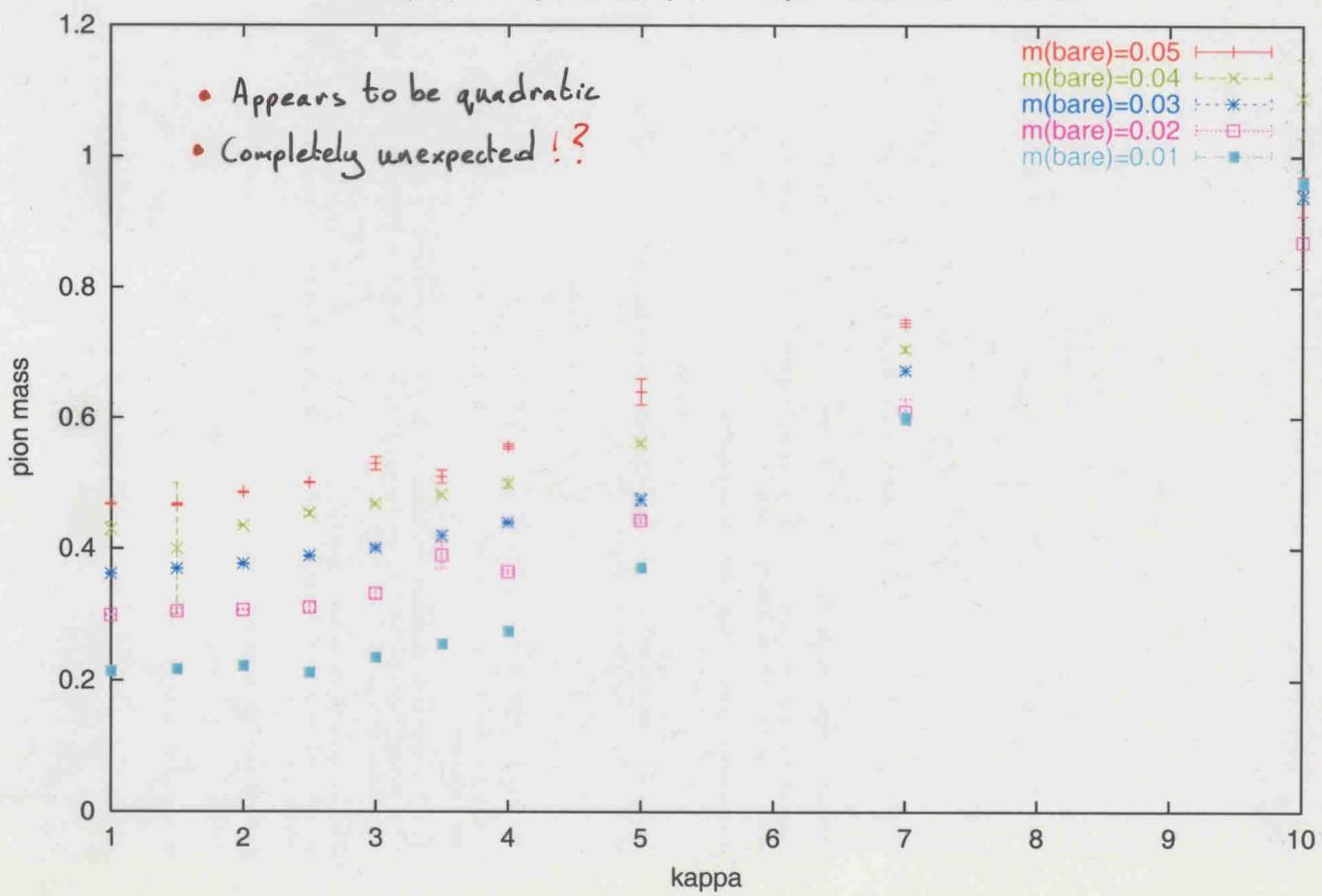
↳ obtained by fitting a cosh curve to the
timeslice propagator

- Renormalized K_S : ratio of the x and y 'masses'

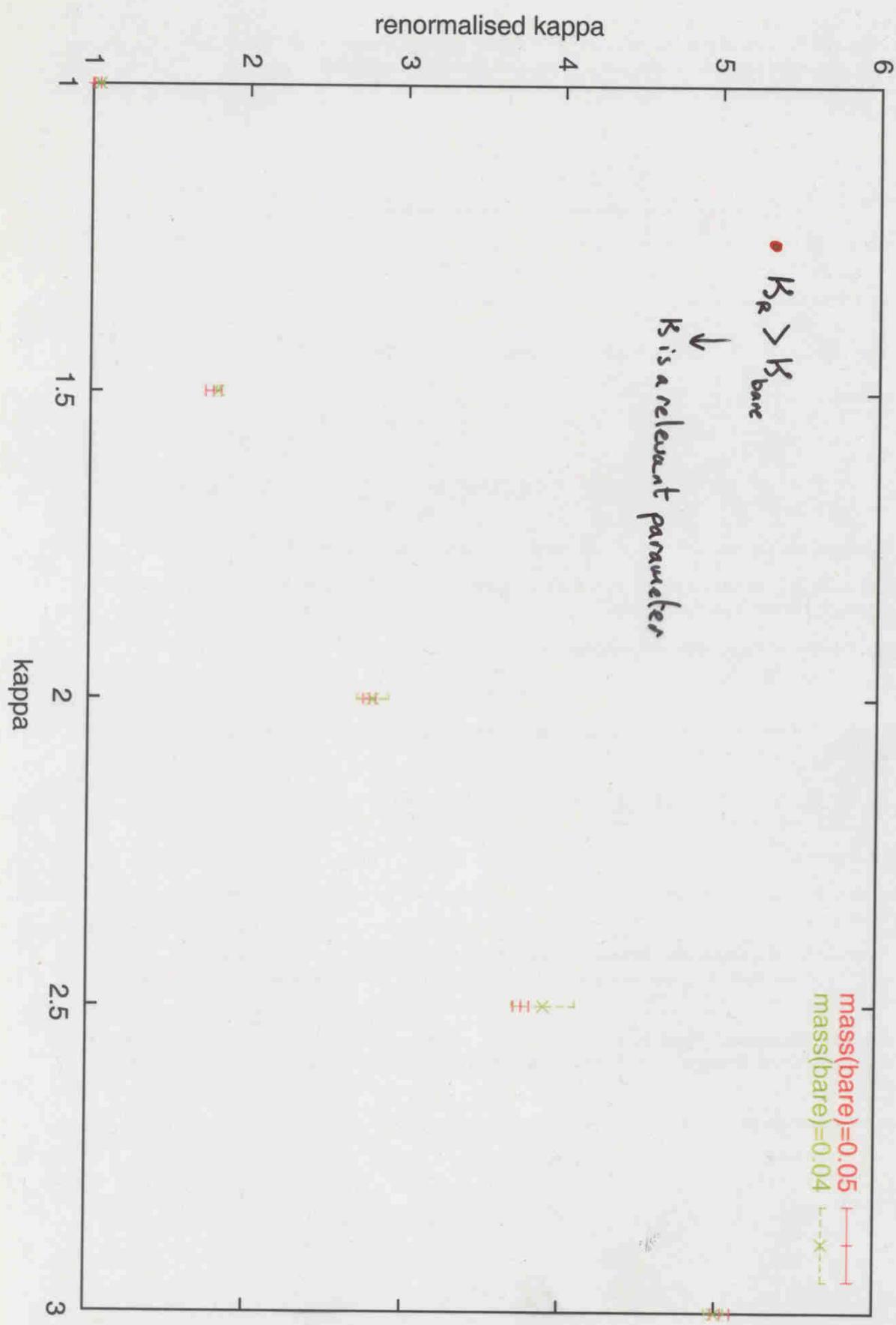
obtained by fitting
cosh curve to
'spaceslice' propagator
in the appropriate direction



Plot of pion mass parameter (t direction) for $\beta=0.2$, 16^3 lattice



Renormalised kappa for beta=0.2



Summary

- Possibility of chiral symmetry restoration as $\beta \rightarrow 10$
 - Perturbative prediction: β is an irrelevant parameter
 - Simulation: it isn't!
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- Need to simulate on larger lattices, with different values of β , take continuum limit etc.
 - Need to examine susceptibilities.